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Unification of Higgs and Gauge Fields in Five Dimensions

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Abstract

We construct realistic theories in which the Higgs fields arise from extra dimensional components of higher dimensional gauge fields. In particular, we present a minimal 5D $SU(3)_C \times SU(3)_W$ model and a unified 5D $SU(6)$ model. In both cases the theory is reduced to the minimal supersymmetric standard model below the compactification scale, with the two Higgs doublets arising from the 5D gauge multiplet. Quarks and Leptons are introduced in the bulk, giving Yukawa couplings without conflicting with higher dimensional gauge invariance. Despite the fact that they arise from higher dimensional gauge interactions, the sizes of these Yukawa couplings can be different from the 4D gauge couplings due to wave-function profiles of the matter zero modes determined by bulk mass parameters. All unwanted fields are made heavy by introducing appropriate matter and superpotentials on branes, which are also the source of intergenerational mixings in the low-energy Yukawa matrices. The theory can accommodate a realistic structure for the Yukawa couplings as well as small neutrino masses. Scenarios for supersymmetry breaking and the μ-term generation are also discussed.

1 Introduction

The unification of seemingly different particles into simplified descriptions remains an important goal in particle physics. Such unification is often attained by symmetries. For instance, unification of quarks and leptons is obtained by assuming non-Abelian gauge symmetries larger than the standard model gauge symmetry [1]. It is also possible to unify fields with different spins if we introduce symmetries relating them. Supersymmetry is an example of such symmetries, although it seems difficult to use it to unify fields in the standard model. Another example is higher dimensional spacetime symmetry. As was shown by Kaluza and Klein, such a symmetry can be used to unify fields which have the same statistics but different spins, such as the graviton and the photon [2].

In this paper we construct realistic theories unifying the Higgs and gauge fields using a higher dimensional spacetime symmetry. The idea of unifying Higgs and gauge fields in higher dimensions is not new. Starting from pioneering work in the late 1970's [3], renewed interest in higher dimensions has resulted in the re-examination of such a possibility [4, 5, 6]. It is, however, not straightforward to construct a completely realistic theory because of the following immediate difficulties:

- It is not trivial to obtain the quartic coupling of the Higgs field, which is required to have successful electroweak symmetry breaking and sufficiently large physical Higgs boson mass.
- It is not easy to obtain Yukawa couplings, since higher dimensional gauge invariance often leads to unwanted massless fields at low energies or vanishing Yukawa couplings.
- Even if we obtain Yukawa couplings, they must have a quite different structure than that of the gauge sector to reproduce the observed quark and lepton masses and mixings. In particular, the Yukawa couplings must have different values for different generations and also intergenerational mixings.

One way of avoiding these problems is to identify the Higgs fields with *scalar fields which are superpartners of the higher dimensional gauge fields*. In this case, we can construct realistic theories unifying the Higgs and gauge fields without encountering the above problems [6]. However, the problems become more severe if we want to identify the Higgs fields with *extra dimensional components of the gauge fields*. Although there are several proposals dealing with these problems (for instance the quartic coupling can be obtained from six dimensional gauge kinetic energies [3] and the Yukawa couplings from non-local operators involving Wilson lines [6]), a complete theory with a realistic phenomenology seems still missing. In this paper we construct a class of realistic theories in which (a part of) the Higgs fields arise from extra dimensional components of higher dimensional gauge fields, without suffering from the above three problems.

We consider higher dimensional supersymmetric gauge theories, which reduce to the minimal supersymmetric standard model (MSSM) below the compactification scale. To obtain the two Higgs doublets from extra dimensional components of the higher dimensional gauge fields, the gauge group in higher dimensions must be larger than the standard model gauge group. This larger gauge group is then broken to the standard model one by compactifying the theory on orbifolds. Such a compactification projects out some of the unwanted fields from low energy theories [7] and leads to special points in the extra dimensions (which we call branes) where the original gauge group is reduced to its subgroup, and on which we can introduce multiplets and interactions respecting only the reduced gauge symmetry [8]. This structure allows us to construct theories with the desired properties. Specifically, we avoid the above three problems in the following way.

- The theory is reduced to the MSSM below the compactification scale, so that the Higgs quartic couplings arise from the D-term potential as in the usual MSSM.
- Although higher dimensional gauge invariance forbids brane-localized Yukawa couplings between the Higgs fields and quark/lepton fields, we can obtain Yukawa couplings from the higher dimensional gauge coupling if we introduce quarks and leptons in the bulk. The potentially-present unwanted massless fields can be made heavy by coupling them to fields located on branes.
- Although the Yukawa couplings arise from the higher dimensional gauge interaction, the low energy Yukawa couplings are in general different from mere gauge couplings due to the presence of the wave-function profiles for the matter fields arising from their bulk masses. Intergenerational mixing can arise from the couplings between the matter fields in the bulk and fields located on branes.

We construct a minimal theory in 5D, in which the higher dimensional gauge group is $SU(3)_C \times SU(3)_W$. We also construct a unified version of the theory: 5D $SU(6)$ model. Although the group theoretical structures of these theories are similar to those of Ref. [6], in our theories (a part of) the Higgs fields arise from extra dimensional components of the higher dimensional gauge field, and not from scalar fields that are superpartners of the gauge field. This allows us to consider a five dimensional theory: the theory with the minimal number of extra dimensions. Extensions to higher dimensional cases are straightforward.

We here note that our mechanism for reproducing realistic Yukawa couplings is quite general and can also be applied to non-supersymmetric theories. This implies that, if we generate the quartic coupling from some other source, for instance from gauge kinetic energies by considering 6D theories, we can construct non-supersymmetric models where the Higgs fields arise from extra dimensional components of the gauge field. The construction should employ a similar gauge symmetry structure, and the values for the low-energy gauge couplings must be reproduced by brane-localized gauge couplings depending on the value for $1/R$ (see section 2). Such a construction could be used to consider theories in which the quadratic divergence for the Higgs doublet is cut off by the size of the compact extra dimensions, if we choose $1/R \sim$ TeV [4].

The organization of the paper is the following. In the next section we present a minimal theory with gauge group $SU(3)_C \times SU(3)_W$. We discuss how the MSSM Yukawa structure is obtained below the compactification scale $1/R$. In section 3 we construct a unified theory based on 5D $SU(6)$. This theory yields the successful prediction of the MSSM for $\sin^2\theta_w$ together with $1/R \sim 10^{16}$ GeV, provided that the volume of the extra dimension is large. Conclusions and discussion are given in section 4.

2 Minimal Theory: 5D $SU(3)_C \times SU(3)_W$ Model

In this section we construct a minimal theory in which the MSSM Higgs doublets arise from extra dimensional components of the gauge fields. We consider a 5D supersymmetric $SU(3)_W$ gauge theory compactified on an S^1/Z_2 orbifold. This $SU(3)_W$ contains the standard model electroweak gauge group: $SU(3)_W \supset SU(2)_L \times U(1)_Y$. The color $SU(3)_C$ interaction can be introduced in a straightforward manner. In subsection 2.1 we illustrate our basic idea, using a single generation model. In subsection 2.2 we generalize it to three generations and discuss how the observed structure of quark and lepton mass matrices is obtained in our model.

2.1 Single generation model

We start by considering the gauge-Higgs sector of the model. The orbifold S^1/Z_2 is constructed by identifying the coordinate of the fifth dimension, $y \in (-\infty, \infty)$, under two operations Z : $y \to -y$ and $Z' : y' \to -y'$ where $y' \equiv y - \pi R$. The resulting space is a line interval $y \in [0, \pi R]$. Under these two operations, various fields can have non-trivial boundary conditions. Using 4D $N = 1$ superfield language, in which the gauge degrees of freedom are contained in $V(A_\mu, \lambda)$ and $\Sigma(\sigma + iA_5, \lambda')$, the boundary conditions for the 5D $SU(3)_W$ gauge multiplet are given by

$$\begin{pmatrix} V \\ \Sigma \end{pmatrix}(x^\mu, -y) = \begin{pmatrix} PVP^{-1} \\ -P\Sigma P^{-1} \end{pmatrix}(x^\mu, y), \qquad \begin{pmatrix} V \\ \Sigma \end{pmatrix}(x^\mu, -y') = \begin{pmatrix} P'VP'^{-1} \\ -P'\Sigma P'^{-1} \end{pmatrix}(x^\mu, y'), \qquad (1)$$

where P and P' are 3×3 matrices acting on gauge space. We now choose P and P' such that $SU(3)_W$ is broken down to $SU(2) \times U(1)$ and the two Higgs doublets are obtained from the 5D gauge multiplet. Specifically, we take $P = P' = \mathrm{diag}(1, 1, -1)$, in which case boundary conditions are given by

$$V: \left(\begin{array}{cc|c} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ \hline (-,-) & (-,-) & (+,+) \end{array} \right), \qquad \Sigma: \left(\begin{array}{cc|c} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ \hline (+,+) & (+,+) & (-,-) \end{array} \right), \tag{2}$$

where the first and second signs represent parities under the two reflections Z and Z', respectively. Since only $(+,+)$ components have zero modes, we find that the gauge group is broken to $SU(2) \times U(1)$ at low energies. We identify this $SU(2) \times U(1)$ as the standard model electroweak gauge group, $SU(2)_L \times U(1)_Y$. We also find that there are two $SU(2)_L$ doublet zero-mode fields arising from Σ, which we identify as the two MSSM Higgs doublets, H_u and H_d. Therefore, at this stage, the low-energy matter content below the compactification scale, $1/R$, is the 4D $N=1$ $SU(2)_L \times U(1)_Y$ gauge multiplet and the two MSSM Higgs doublets. Since the low-energy theory below $1/R$ is 4D $N=1$ supersymmetric, the Higgs quartic couplings arise from the D-term potential as in the usual MSSM.

We next consider the 5D gauge symmetry structure of the theory. Although the gauge symmetry is broken to $SU(2)_L \times U(1)_Y$ in the low-energy 4D theory, the original 5D theory has a larger gauge symmetry. We find that this gauge symmetry is $SU(3)_W$ but with the gauge transformation parameters obeying the same boundary conditions as the corresponding 4D gauge fields. Specifically, the $SU(2)_L \times U(1)_Y$ gauge parameters have $(Z, Z') = (+,+)$ parities, while $SU(3)_W/(SU(2)_L \times U(1)_Y)$ ones have $(Z, Z') = (-,-)$. This implies that gauge transformation parameters for $SU(3)_W/(SU(2)_L \times U(1)_Y)$ always vanish at $y=0$ and πR, so that the gauge symmetry on these fixed points (branes) is reduced to $SU(2)_L \times U(1)_Y$; in particular, we can introduce fields and interactions that respect only $SU(2)_L \times U(1)_Y$ on these branes [8]. This position-dependent gauge symmetry structure is very important for constructing our theory.

What is the compactification scale $1/R$? If the 4D gauge couplings arose entirely from the 5D bulk gauge coupling, there would be a relation between the zero-mode gauge couplings of $SU(2)_L$ and $U(1)_Y$ at the scale of $1/R$. Denoting the $SU(2)_L$ coupling and the conventionally normalized hypercharge coupling as g_2 and g_Y, respectively, this relation is given by $g_Y = \sqrt{3} g_2$. Here, we have neglected the difference between the cutoff scale M_* and the compactification scale $1/R$, which could slightly affect the relation. Assuming the MSSM matter content below $1/R$, this would require $1/R$ to be much larger than the Planck scale for low energy data to be reproduced. However, the 4D gauge couplings can also receive contributions from brane-localized gauge kinetic terms, such as $\delta(y)\lambda_0 F_{\mu\nu}^2$ and $\delta(y-\pi R)\lambda_\pi F_{\mu\nu}^2$, which can have different coefficients for $SU(2)_L$ and $U(1)_Y$. If the volume of the extra dimension is not large, these terms are expected to give non-negligible contributions to the 4D gauge couplings. In this case, we do not have any definite relation between g_2 and g_Y at the scale of $1/R$, and consequently the value of $1/R$ is not constrained by the low-energy gauge couplings. Here we simply choose brane-localized terms such that low energy data for g_2 and g_Y are reproduced, and treat $1/R$ as a free parameter. (The situation is quite different in the unified model given in section 3.) From now on, we normalize the generator of $U(1)_Y \subset SU(3)_W$ to match the conventional definition of hypercharge: $T_Y = \mathrm{diag}(1/6, 1/6, -1/3)$, (so that the Higgs doublets have hypercharges $\pm 1/2$).

The color $SU(3)_C$ interaction can be added in a straightforward way. We introduce the 5D $SU(3)_C$ gauge multiplet in the bulk: $\{V_C, \Sigma_C\}$. The Z and Z' parities are assigned as $V_C(+,+)$ and $\Sigma_C(-,-)$, giving the 4D $N=1$ $SU(3)_C$ gauge multiplet below $1/R$. Obviously, the Higgs doublets are singlet under $SU(3)_C$ as it should be. The gauge symmetry in the bulk is now $SU(3)_C \times SU(3)_W$ and that on the two branes is $SU(3)_C \times SU(2)_L \times U(1)_Y$.

Having understood the gauge-Higgs sector, we now consider matter fields. We first note that it is not trivial to write down Yukawa couplings as usual local operators. One might naively think that we can introduce quark and lepton chiral supermultiplets on a brane and couple them to the Higgs fields through brane-localized Yukawa couplings. However, it turns out that this does not work. The source of the difficulty is the gauge transformation property of the Higgs fields. Using the 4D $N=1$ superfield language, the gauge transformation of the 5D $SU(3)_W$ gauge multiplet is given by

$$e^V \;\to\; e^{\Lambda} e^V e^{\Lambda^\dagger}, \tag{3}$$

$$\Sigma \;\to\; e^{\Lambda}(\Sigma - \sqrt{2}\partial_y) e^{-\Lambda}, \tag{4}$$

where Λ is a chiral superfield containing a gauge transformation parameter α [9]. Since the Higgs fields are identified with components of Σ, we find that they transform non-linearly under the 5D gauge transformation. This prevents us to write a Yukawa coupling to the matter fields on the brane. (In component language, $A_5 \to A_5 + \partial_y \alpha + \cdots$ forbids the Yukawa coupling $\mathcal{L} \sim \delta(y) q \bar{q} A_5$.) Therefore, we choose to introduce quarks and leptons in the bulk, and produce Yukawa couplings from the 5D gauge interaction.

We begin with the down-type quark sector. We consider a hypermultiplet $\{\mathcal{D}, \mathcal{D}^c\}$ transforming as $\mathbf{3}$ under both $SU(3)_C$ and $SU(3)_W$, where $\mathcal{D}$ and $\mathcal{D}^c$ represent 4D $N=1$ chiral superfields. In our notation, a conjugated field has the opposite transformation property with the non-conjugated field, and we specify the transformation property of a hypermultiplet by that of the non-conjugated chiral superfield; for instance, $\mathcal{D}$ and $\mathcal{D}^c$ transform as $\mathbf{3}$ and $\mathbf{3}^*$ under $SU(3)_C$, respectively. We choose the boundary conditions for this hypermultiplet as follows:

$$\mathcal{D} \;=\; \mathcal{D}_Q^{(+,+)}(\mathbf{3},\mathbf{2})_{1/6} \oplus \mathcal{D}_D^{(-,-)}(\mathbf{3},\mathbf{1})_{-1/3}, \tag{5}$$

$$\mathcal{D}^c \;=\; \mathcal{D}_Q^{c\,(-,-)}(\mathbf{3}^*,\mathbf{2})_{-1/6} \oplus \mathcal{D}_D^{c\,(+,+)}(\mathbf{3}^*,\mathbf{1})_{1/3}, \tag{6}$$

where the superscripts denote transformation properties under (Z, Z'), and the numbers with parentheses represent gauge quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y$ with hypercharges normalized conventionally. Since only $(Z, Z') = (+,+)$ components have zero modes, we find that there are only two zero modes, which arise from $\mathcal{D}_Q(\mathbf{3},\mathbf{2})_{1/6}$ and $\mathcal{D}^c_D(\mathbf{3}^*,\mathbf{1})_{1/3}$.

What is the gauge interaction for this hypermultiplet? Using the 4D $N=1$ superfield language, the 5D gauge interaction is written as [9]

$$S = \int d^4x\, dy \left[\int d^2\theta\, d^2\bar\theta \left(\mathcal{D}^\dagger e^{-V} \mathcal{D} + \mathcal{D}^c e^{V} \mathcal{D}^{c\dagger} \right) + \left(\int d^2\theta\, \mathcal{D}^c (\partial_y - \Sigma) \mathcal{D} + \text{h.c.} \right) \right]. \tag{7}$$

At low energies, the first two terms give the usual 4D $N=1$ gauge interaction for the zero modes of $\mathcal{D}_Q$ and $\mathcal{D}^c_D$. On the other hand, the third (superpotential) term gives the interaction among the zero modes of $\mathcal{D}_Q$, $\mathcal{D}^c_D$ and Σ:

$$S = \int d^4x \int d^2\theta\; y'_d \mathcal{D}^c_D H_d \mathcal{D}_Q + \text{h.c.}, \tag{8}$$

where y'_d is the coupling constant and H_d represents the $(\mathbf{1},\mathbf{2})_{-1/2}$ component of Σ. This has the form of the Yukawa coupling for the down-type quark. Therefore, we are tempted to identify the zero modes of $\mathcal{D}_Q$ and $\mathcal{D}^c_D$ as the MSSM quark supermultiplets Q and D. Before making this identification, however, we have to consider the up-type quark sector, where we will learn that the actual identification must be somewhat more subtle.

For the up-type quarks, we introduce a hypermultiplet $\{\mathcal{U}, \mathcal{U}^c\}$ transforming as $\mathbf{3}^*$ and $\mathbf{6}$ under $SU(3)_C$ and $SU(3)_W$. The boundary conditions for this hypermultiplet are chosen as

$$\begin{aligned} \mathcal{U} &= \mathcal{U}_T^{(+,+)}(\mathbf{3}^*,\mathbf{3})_{1/3} \oplus \mathcal{U}_Q^{(-,-)}(\mathbf{3}^*,\mathbf{2})_{-1/6} \oplus \mathcal{U}_U^{(+,+)}(\mathbf{3}^*,\mathbf{1})_{-2/3}, && (9) \\ \mathcal{U}^c &= \mathcal{U}_T^{c\,(-,-)}(\mathbf{3},\mathbf{3})_{-1/3} \oplus \mathcal{U}_Q^{c\,(+,+)}(\mathbf{3},\mathbf{2})_{1/6} \oplus \mathcal{U}_U^{c\,(-,-)}(\mathbf{3},\mathbf{1})_{2/3}. && (10) \end{aligned}$$

Thus, we have zero modes for $\mathcal{U}_T(\mathbf{3}^*,\mathbf{3})_{1/3}$, $\mathcal{U}_U(\mathbf{3}^*,\mathbf{1})_{-2/3}$ and $\mathcal{U}^c_Q(\mathbf{3},\mathbf{2})_{1/6}$. As in the case of the $\{\mathcal{D},\mathcal{D}^c\}$ hypermultiplet, the 5D gauge interaction reproduces, at low energies, the Yukawa couplings among these zero modes and the zero mode of Σ, of the form

$$S = \int d^4x \int d^2\theta\, \left(y'_u \mathcal{U}^c_Q H_u \mathcal{U}_U + y''_u \mathcal{U}^c_Q H_d \mathcal{U}_T \right) + \text{h.c.}, \tag{11}$$

where y'_u and y''_u are coupling constants and H_u represents the $(\mathbf{1},\mathbf{2})_{1/2}$ component of Σ. The first term appears the up-type Yukawa coupling. However, here we encounter a few problems. First, after canonically normalizing the 4D fields, we find that y'_u and y'_d have the same value as the gauge coupling that would arise purely from the 5D bulk gauge coupling: $y'_u = y'_d = g$, where g is expected to be similar in size with the $SU(2)_L$ and $U(1)_Y$ gauge couplings (y''_u is also equal to g). This is grossly incompatible with observation, especially for the first generation. Second, the quark doublets, $\mathcal{U}^c_Q$ and $\mathcal{D}_Q$, appearing in the up-type and down-type Yukawa couplings are different fields, while the two must be an identical field in the MSSM. Third, we have an unwanted massless field which does not appear in the MSSM: the zero mode of $\mathcal{U}_T(\mathbf{3}^*, \mathbf{3})_{1/3}$. Below we will address these issues in turn.

The first problem, $y'_u = y'_d = g$, can be solved by introducing bulk masses for the hypermultiplets:

$$S = \int d^4x \, dy \left[\int d^2\theta \, (M_u \mathcal{U}^c \mathcal{U} + M_d \mathcal{D}^c \mathcal{D}) + \text{h.c.} \right], \tag{12}$$

where M_u and M_d are real. (In the covering space, these masses are odd under $y \to -y$: they are M for $0 < y < \pi R$ but $-M$ for $-\pi R < y < 0$.) With these bulk masses, wave-functions for the zero-mode fields have exponential profiles in the extra dimension. As an example, here we choose $M_u > 0$ and $M_d < 0$. In this case the wave-functions for the zero modes of $\mathcal{U}^c_Q$, $\mathcal{U}_U$, $\mathcal{D}_Q$, and $\mathcal{D}^c_D$ have profiles as $\exp\{-|M_u|y\}$, $\exp\{|M_u|(y - \pi R)\}$, $\exp\{-|M_d|y\}$, and $\exp\{|M_d|(y - \pi R)\}$, respectively. The zero modes for $\mathcal{U}^c_Q$ and $\mathcal{D}_Q$ are localized toward the $y = 0$ brane, while those of $\mathcal{U}_U$ and $\mathcal{D}^c_D$ toward the $y = \pi R$ brane (the zero mode of $\mathcal{U}_T$ is localized to the $y = \pi R$ brane). Since the 4D "Yukawa couplings", y'_u and y'_d, are proportional to the overlap of the zero-mode wave-functions, they now differ from the 4D "gauge coupling", g:

$$y'_u = \frac{\pi R |M_u| g}{\sinh(\pi R |M_u|)} \stackrel{|M_u|R \gtrsim 1}{\longrightarrow} 2\pi R |M_u| e^{-\pi R |M_u|} g, \tag{13}$$

$$y'_d = \frac{\pi R |M_d| g}{\sinh(\pi R |M_d|)} \stackrel{|M_d|R \gtrsim 1}{\longrightarrow} 2\pi R |M_d| e^{-\pi R |M_d|} g. \tag{14}$$

Therefore, we can choose these couplings to be free parameters of the theory (the coupling y''_d is equal to y'_d). An important point is that they are exponentially suppressed for large bulk masses, and this fact will be used in the next subsection for generating the hierarchy of quark and lepton masses.

We now consider the second and third problems. Regarding the third problem, the unwanted $\mathcal{U}_T$ field, we introduce a chiral superfield $\bar{\mathcal{U}}_{\bar{T}}(\mathbf{3}, \mathbf{3})_{-1/3}$ on the $y = \pi R$ brane, which has the opposite transformation property with $\mathcal{U}_T$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$. Remember that only the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry is active on the brane, and we can introduce an arbitrary $SU(3)_C \times SU(2)_L \times U(1)_Y$ representation, which does not have to be in a representation of $SU(3)_W$.[1] Then, by introducing a brane mass term $\delta(y - \pi R)[\kappa_T \bar{\mathcal{U}}_{\bar{T}} \mathcal{U}_T]_{\theta^2}$, we can make the unwanted field, $\mathcal{U}_T$, heavy (together with the new field, $\bar{\mathcal{U}}_{\bar{T}}$). The mass of these fields is naturally expected to be $1/R$ or higher.

The second problem can be dealt with in a similar way. We introduce a chiral superfield

[1]This implies that $U(1)_Y$ charges of brane matter do not necessarily have to be quantized in units of the bulk non-Abelian gauge group. We do not address this issue of quantization of brane $U(1)$ charges in this paper. One possibility of obtaining the desired quantization is to consider higher dimensional theories with a larger gauge group, as discussed in Ref. [10].

$\bar{Q}(\mathbf{3}^*, \mathbf{2})_{-1/6}$ on the $y=0$ brane, together with the superpotential term

$$S = \int d^4x \; dy \; \delta(y) \left[\int d^2\theta \; \bar{Q} (\kappa_{Q,1} \mathcal{U}_Q^c + \kappa_{Q,2} \mathcal{D}_Q) + \text{h.c.} \right]. \tag{15}$$

This makes one linear combination of $\mathcal{U}_Q^c$ and $\mathcal{D}_Q$ heavy, of mass $1/R$ or higher, together with $\bar{Q}$. We define this linear combination as $Q_H \equiv \cos\phi_Q \, \mathcal{U}_Q^c + \sin\phi_Q \, \mathcal{D}_Q$, where $\tan\phi_Q = \kappa_{Q,2}/\kappa_{Q,1}$. Then, we find that the orthogonal combination, $Q \equiv -\sin\phi_Q \, \mathcal{U}_Q^c + \cos\phi_Q \, \mathcal{D}_Q$, remains massless at low energies. We identify this field as the quark doublet of the MSSM. Therefore, we finally obtain the following field content below $1/R$: the 4D $N=1$ $SU(3)_C \times SU(2)_L \times U(1)_Y$ vector supermultiplets, two Higgs chiral superfields, $H_u(\mathbf{1}, \mathbf{2})_{1/2}$ and $H_d(\mathbf{1}, \mathbf{2})_{-1/2}$, and three quark chiral superfields, $Q(\mathbf{3}, \mathbf{2})_{1/6}$, $U \equiv \mathcal{U}_U(\mathbf{3}^*, \mathbf{1})_{-2/3}$ and $D \equiv \mathcal{D}_D^c(\mathbf{3}^*, \mathbf{1})_{1/3}$. They have usual 4D $N=1$ gauge interactions as well as the Yukawa couplings

$$S = \int d^4x \int d^2\theta \; (y_u Q U H_u + y_d Q D H_d) + \text{h.c.}, \tag{16}$$

where $y_u = -y'_u \sin\phi_Q$ and $y_d = y'_d \cos\phi_Q$. This is exactly the quark sector of the MSSM. Thus we find that our theory, in which the Higgs fields arise as an extra dimensional component of the 5D gauge field, reduces to the MSSM at energies below $1/R$, as far as the quark sector is concerned.

At this point we make one comment. Since the form of Eqs. (13, 14) implies $y'_u, y'_d \leq g$ and thus $|y_u|, |y_d| \leq g$, one may worry that the top quark mass is not reproduced in our theory. However, this is not necessarily the case. First, g is not trivially related to the observed gauge coupling values; these relations can involve unknown contributions from brane-localized gauge kinetic terms, so that g can be larger than the weak gauge couplings. Second, the expressions for the Yukawa couplings given above apply at the scale of $1/R$. In fact, in the unified model given in the next section, $1/R$ is around the conventional unified mass scale, *i.e.* $1/R \sim 10^{16}$ GeV, and g is the unified gauge coupling, $g \sim 0.7$. In this case our theory requires $y_t \lesssim 0.7$ at $1/R$, but this is not in contradiction with the observed value of the top quark mass.

The lepton sector can be worked out similarly. We first consider charged leptons. We introduce a hypermultiplet $\{\mathcal{E}, \mathcal{E}^c\}$ transforming as $\mathbf{1}$ and $\mathbf{10}$ under $SU(3)_C$ and $SU(3)_W$. The boundary conditions are chosen as

$$\mathcal{E} = \mathcal{E}_Q^{(+,+)}(\mathbf{1}, \mathbf{4})_{1/2} \oplus \mathcal{E}_T^{(-,-)}(\mathbf{1}, \mathbf{3})_0 \oplus \mathcal{E}_L^{(+,+)}(\mathbf{1}, \mathbf{2})_{-1/2} \oplus \mathcal{E}_E^{(-,-)}(\mathbf{1}, \mathbf{1})_{-1}, \tag{17}$$

$$\mathcal{E}^c = \mathcal{E}_Q^{c\,(-,-)}(\mathbf{1}, \mathbf{4})_{-1/2} \oplus \mathcal{E}_T^{c\,(+,+)}(\mathbf{1}, \mathbf{3})_0 \oplus \mathcal{E}_L^{c\,(-,-)}(\mathbf{1}, \mathbf{2})_{1/2} \oplus \mathcal{E}_E^{c\,(+,+)}(\mathbf{1}, \mathbf{1})_1. \tag{18}$$

The zero-mode fields arise from $\mathcal{E}_Q(\mathbf{1}, \mathbf{4})_{1/2}$, $\mathcal{E}_L(\mathbf{1}, \mathbf{2})_{-1/2}$, $\mathcal{E}_T^c(\mathbf{1}, \mathbf{3})_0$ and $\mathcal{E}_E^c(\mathbf{1}, \mathbf{1})_1$. Introducing a bulk hypermultiplet mass M_e, which we assume to be positive for simplicity, the zero modes of

$\mathcal{E}_Q$ and $\mathcal{E}_L$ ($\mathcal{E}_T^c$ and $\mathcal{E}_E^c$) are localized toward the $y = \pi R$ ($y = 0$) brane. The 5D gauge interaction yields the Yukawa coupling of the form

$$S = \int d^4x \int d^2\theta \; (y'_e \mathcal{E}_L H_d \mathcal{E}_E^c + \cdots) + \text{h.c.}, \tag{19}$$

where y'_e is given as Eq. (13) with $y'_u \to y'_e$ and $M_u \to M_e$. The unwanted fields, $\mathcal{E}_Q$ and $\mathcal{E}_T^c$, can be made heavy by introducing brane-localized chiral supermultiplets, $\bar{\mathcal{E}}_{\bar{Q}}$ on the $y = \pi R$ brane and $\bar{\mathcal{E}}_{\bar{T}}^c$ on the $y = 0$ brane, together with the brane superpotential terms $\delta(y - \pi R)[\mathcal{E}_Q \bar{\mathcal{E}}_{\bar{Q}}]_{\theta^2}$ and $\delta(y)[\mathcal{E}_T^c \bar{\mathcal{E}}_{\bar{T}}^c]_{\theta^2}$. Then, if we define $L \equiv \mathcal{E}_L$ and $E \equiv \mathcal{E}_E^c$, we find that the term in Eq. (19) gives the charged-lepton Yukawa coupling in the MSSM (this identification must be slightly modified when we consider neutrino masses, see below).

Small neutrino masses are introduced as follows. To employ the conventional seesaw mechanism [11], we introduce a hypermultiplet $\{\mathcal{N}, \mathcal{N}^c\}$ transforming as $\mathbf{1}$ and $\mathbf{8}$ under $SU(3)_C$ and $SU(3)_W$, respectively. The boundary conditions are given by

$$\mathcal{N} = \mathcal{N}_T^{(+,+)}(\mathbf{1},\mathbf{3})_0 \oplus \mathcal{N}_L^{(-,-)}(\mathbf{1},\mathbf{2})_{1/2} \oplus \mathcal{N}_H^{(-,-)}(\mathbf{1},\mathbf{2})_{-1/2} \oplus \mathcal{N}_N^{(+,+)}(\mathbf{1},\mathbf{1})_0, \tag{20}$$

$$\mathcal{N}^c = \mathcal{N}_T^{c\,(-,-)}(\mathbf{1},\mathbf{3})_0 \oplus \mathcal{N}_L^{c\,(+,+)}(\mathbf{1},\mathbf{2})_{-1/2} \oplus \mathcal{N}_H^{c\,(+,+)}(\mathbf{1},\mathbf{2})_{1/2} \oplus \mathcal{N}_N^{c\,(-,-)}(\mathbf{1},\mathbf{1})_0. \tag{21}$$

The zero modes then arise from $\mathcal{N}_T(\mathbf{1},\mathbf{3})_0$, $\mathcal{N}_N(\mathbf{1},\mathbf{1})_0$, $\mathcal{N}_L^c(\mathbf{1},\mathbf{2})_{-1/2}$ and $\mathcal{N}_H^c(\mathbf{1},\mathbf{2})_{1/2}$. The bulk hypermultiplet mass M_n, which we take to be negative, is introduced as before, localizing the zero modes of $\mathcal{N}_T$ and $\mathcal{N}_N$ ($\mathcal{N}_L^c$ and $\mathcal{N}_H^c$) to the $y = 0$ ($y = \pi R$) brane. At low energies, the 5D gauge interaction yields

$$S = \int d^4x \int d^2\theta \; (y'_n \mathcal{N}_N H_u \mathcal{N}_L^c + \cdots) + \text{h.c.}, \tag{22}$$

where y'_n is given as Eq. (13) with $y'_u \to y'_n$ and $M_u \to M_n$.

Now we find that the situation is similar to the quark case. We consider hypermultiplets $\{\mathcal{E}, \mathcal{E}^c\}$ and $\{\mathcal{N}, \mathcal{N}^c\}$, with the boundary conditions given by Eqs. (17, 18, 20, 21). Among the zero modes, $\mathcal{E}_Q$, $\mathcal{E}_T^c$, $\mathcal{N}_T$ and $\mathcal{N}_H^c$ fields are made heavy, of mass around $1/R$, by introducing appropriate brane fields and superpotentials: $\delta(y - \pi R)[\mathcal{E}_Q \bar{\mathcal{E}}_{\bar{Q}}]_{\theta^2}$, $\delta(y)[\mathcal{E}_T^c \bar{\mathcal{E}}_{\bar{T}}^c]_{\theta^2}$, $\delta(y)[\mathcal{N}_T \bar{\mathcal{N}}_{\bar{T}}]_{\theta^2}$ and $\delta(y - \pi R)[\mathcal{N}_H^c \bar{\mathcal{N}}_{\bar{H}}^c]_{\theta^2}$. We also introduce a brane chiral superfield $\bar{L}(\mathbf{1},\mathbf{2})_{1/2}$ on the $y = \pi R$ brane, together with the superpotential

$$S = \int d^4x \; dy \; \delta(y - \pi R) \left[\int d^2\theta \; \bar{L}(\kappa_{L,1} \mathcal{E}_L + \kappa_{L,2} \mathcal{N}_L^c) + \text{h.c.} \right]. \tag{23}$$

This makes one linear combination of $\mathcal{E}_L$ and $\mathcal{N}_L^c$, $L_H \equiv \cos\phi_L \, \mathcal{E}_L + \sin\phi_L \, \mathcal{N}_L^c$, heavy, where $\tan\phi_L = \kappa_{L,2}/\kappa_{L,1}$. Thus, at energies below $1/R$, we have three chiral superfields: $L(\mathbf{1},\mathbf{2})_{-1/2} \equiv -\sin\phi_L \, \mathcal{E}_L + \cos\phi_L \, \mathcal{N}_L^c$, $E \equiv \mathcal{E}_E^c(\mathbf{1},\mathbf{1})_1$ and $N \equiv \mathcal{N}_N(\mathbf{1},\mathbf{1})_0$. Introducing the brane superpotential $\delta(y)[(\kappa_N/2)\mathcal{N}_N^2]_{\theta^2}$, we find that these fields have the following superpotential:

$$S = \int d^4x \int d^2\theta \; \left(y_e L E H_d + y_n L N H_u + \frac{M_R}{2} N^2 \right) + \text{h.c.}, \tag{24}$$

where $y_e = -y'_e \sin\phi_L$, $y_n = y'_n \cos\phi_L$ and $M_R = 2\kappa_N |M_n|/(1 - e^{-2\pi R |M_n|})$. Since we expect M_R to be large, $M_R \sim M_n \sim 1/R$, this superpotential gives small neutrino masses through the seesaw mechanism, as well as the charged-lepton Yukawa coupling.

Finally, we comment on anomalies. Since the field content of our theory below $1/R$ is that of the MSSM (with right-handed neutrino), there are no 4D gauge anomalies. There could still be anomalies in 5D localized on the two branes, which are equal and opposite. However, we can always cancel these anomalies by introducing a bulk Chern-Simons term, recovering the consistency of the theory [12].

2.2 Three generation model

In this subsection, we generalize the above single generation model to a realistic three generation model. The basic idea is the same. We consider the 5D $SU(3)_C \times SU(3)_W$ supersymmetric gauge theory, with the boundary conditions for the gauge fields given as in the previous subsection. Below $1/R$, this yields the 4D $N = 1$ $SU(3)_C \times SU(2)_L \times U(1)_Y$ vector superfields, together with the two Higgs chiral superfields, H_u and H_d, arising from Σ of $SU(3)_W$. The 5D gauge symmetry structure is given as before: the bulk has $SU(3)_C \times SU(3)_W$ while the branes have only $SU(3)_C \times SU(2)_L \times U(1)_Y$.

We start with the quark sector. We consider three down-type hypermultiplets, $\{\mathcal{D}_i, \mathcal{D}^c_i\}$ ($i = 1, 2, 3$), transforming as $(\mathbf{3}, \mathbf{3})$ under $SU(3)_C \times SU(3)_W$, and three up-type hypermultiplets, $\{\mathcal{U}_i, \mathcal{U}^c_i\}$ ($i = 1, 2, 3$), transforming as $(\mathbf{3}^*, \mathbf{6})$ under $SU(3)_C \times SU(3)_W$. We introduce bulk masses $M_{u,i}$ and $M_{d,i}$ for each hypermultiplet, which we choose $M_{u,i} > 0$ and $M_{d,i} < 0$. These hypermultiplets obey the boundary conditions as in Eqs. (5, 6, 9, 10). Among the resulting zero modes, those arising from $\mathcal{U}_{T,i}$ are made heavy by coupling to three brane fields $\bar{\mathcal{U}}_{\bar{T},i}$ on the $y = \pi R$ brane: $\delta(y - \pi R)[\kappa_{T,ij} \bar{\mathcal{U}}_{\bar{T},i} \mathcal{U}_{T,j}]_{\theta^2}$ with $\mathrm{rank}(\kappa_{T,ij}) = 3$. Below we concentrate on the rest of the zero modes, $\mathcal{D}_{Q,i}$, $\mathcal{D}^c_{D,i}$, $\mathcal{U}_{U,i}$ and $\mathcal{U}^c_{Q,i}$, and see how the observed structure of the quark mass matrices is obtained in our model.

We first consider the superpotential Yukawa terms that arise directly from the 5D gauge interaction. They are given by

$$S = \int d^4x \int d^2\theta \sum_{i=1}^{3} \left(y'_{u,i} \mathcal{U}^c_{Q,i} \mathcal{U}_{U,i} H_u + y'_{d,i} \mathcal{D}_{Q,i} \mathcal{D}^c_{D,i} H_d \right) + \text{h.c.}, \tag{25}$$

where $y'_{u,i}$ and $y'_{d,i}$ are given by Eqs. (13, 14) with $(y'_u, M_u) \to (y'_{u,i}, M_{u,i})$ and $(y'_d, M_d) \to (y'_{d,i}, M_{d,i})$, which we treat as free parameters of the theory. Since these interactions arise from a part of the 5D gauge interaction, they are diagonal in flavor space. The intergenerational mixing then must come from the brane-localized mass terms required to make the unwanted zero-mode fields heavy.

The brane superpotential making the unwanted zero-mode fields heavy is now given as

$$S = \int d^4x \; dy \; \delta(y) \left[\int d^2\theta \sum_{i,j=1}^{3} \left(\eta_{ij} \mathcal{U}^c_{Q,i} + \lambda_{ij} \mathcal{D}_{Q,i} \right) \bar{Q}_j + \text{h.c.} \right], \tag{26}$$

where $\bar{Q}_i(\mathbf{3}^*, \mathbf{2})_{-1/6}$ are chiral superfields localized on the $y=0$ brane. This yields the superpotential mass term between the zero-mode and the brane-localized fields

$$W_M = \left(\begin{array}{c|c} \mathcal{U}^c_Q & \mathcal{D}_Q \end{array} \right) \left(\begin{array}{c} M_\eta \\ \hline M_\lambda \end{array} \right) \bar{Q}. \tag{27}$$

Here, we have used a matrix notation: $\mathcal{U}^c_Q$ and $\mathcal{D}_Q$ ($\bar{Q}$) represent 3-dimensional row (column) vectors, and M_η and M_λ are 3×3 matrices. We can diagonalize this mass term by rotating the fields by 6×6 and 3×3 unitary matrices, $U^Q_{6\times 6}$ and $U^{\bar{Q}}_{3\times 3}$,

$$\left(\begin{array}{c|c} \mathcal{U}^c_Q & \mathcal{D}_Q \end{array} \right) \equiv \left(\begin{array}{c|c} Q_H & Q \end{array} \right) U^Q_{6\times 6} \equiv \left(\begin{array}{c|c} Q_H & Q \end{array} \right) \left(\begin{array}{c|c} U^{Q(1)}_{3\times 3} & U^{Q(2)}_{3\times 3} \\ \hline U^{Q(3)}_{3\times 3} & U^{Q(4)}_{3\times 3} \end{array} \right), \qquad \bar{Q} \equiv U^{\bar{Q}}_{3\times 3} \bar{Q}', \tag{28}$$

as

$$W_M = \left(\begin{array}{c|c} Q_H & Q \end{array} \right) U^Q_{6\times 6} \left(\begin{array}{c} M_\eta \\ \hline M_\lambda \end{array} \right) U^{\bar{Q}}_{3\times 3} \bar{Q}' = \left(\begin{array}{c|c} Q_H & Q \end{array} \right) \left(\begin{array}{c} M_{\rm diag} \\ \hline \mathbf{0}_{3\times 3} \end{array} \right) \bar{Q}', \tag{29}$$

where Q_H and Q ($\bar{Q}'$) are 3-dimensional row (column) vectors and $M_{\rm diag}$ is a diagonal 3×3 matrix. Therefore, assuming $\text{rank}(M_{\rm diag}) = 3$, we find that three linear combinations, Q_H's, of $\mathcal{U}^c_{Q,i}$ and $\mathcal{D}_{Q,i}$ become heavy together with the brane fields $\bar{Q}_i$, and only the other three linear combinations, Q's, remain massless below $1/R$. We identify these modes as the quark-doublet superfields of the MSSM and work out the resulting structure for the Yukawa couplings.

The $SU(2)_L$-singlet quark superfields of the MSSM are identified as $U_i \equiv \mathcal{U}_{U,i}$ and $D_i \equiv \mathcal{D}^c_{D,i}$. Then, we find from Eq. (25) that the low-energy 4D Yukawa couplings are given by

$$S = \int d^4x \int d^2\theta \; \left(Q \, U^{Q(3)}_{3\times 3} Y'_u \, U H_u + Q \, U^{Q(4)}_{3\times 3} Y'_d \, D H_d \right) + \text{h.c.}, \tag{30}$$

where we have used a matrix notation: U and D represent 3-dimensional column vectors, $Y'_u \equiv \text{diag}(y'_{u,1}, y'_{u,2}, y'_{u,3})$, and $Y'_d \equiv \text{diag}(y'_{d,1}, y'_{d,2}, y'_{d,3})$. We thus find that the MSSM Yukawa matrices, Y_u and Y_d, are given by

$$Y_u = U^{Q(3)}_{3\times 3} Y'_u, \tag{31}$$
$$Y_d = U^{Q(4)}_{3\times 3} Y'_d, \tag{32}$$

in our theory (these Yukawa matrices should be viewed as the running couplings at the scale of $1/R$). This structure is sufficiently general to accommodate the observed quark masses and mixings. The quark masses are obtained by diagonalizing the Yukawa matrices as

$$V^\dagger_{uL} \, U^{Q(3)}_{3\times 3} Y'_u \langle H_u \rangle \, V_{uR} = \text{diag}(m_u, m_c, m_t), \tag{33}$$
$$V^\dagger_{dL} \, U^{Q(4)}_{3\times 3} Y'_d \langle H_d \rangle \, V_{dR} = \text{diag}(m_d, m_s, m_b), \tag{34}$$

where V_{uL}, V_{uR}, V_{dL}, and V_{dR} are unitary matrices. The CKM matrix is then given by

$$V_{\rm CKM} = V_{uL}^{\dagger} V_{dL}. \tag{35}$$

It is interesting to note that the elements in the matrices Y'_u and Y'_d have exponential sensitivity to the bulk masses of matter hypermultiplets and could naturally be the source of hierarchies for the quark masses. Note that the bulk hypermultiplet masses have also been used to generate fermion mass hierarchies in different contexts [13].

The lepton sector works quite similarly. We introduce three charged-lepton hypermultiplets, $\{\mathcal{E}_i, \mathcal{E}_i^c\}$, transforming as $(\mathbf{1}, \mathbf{10})$ under $SU(3)_C \times SU(3)_W$, and three neutrino hypermultiplets, $\{\mathcal{N}_i, \mathcal{N}_i^c\}$, transforming as $(\mathbf{1}, \mathbf{8})$ under $SU(3)_C \times SU(3)_W$. The boundary conditions are given by Eqs. (17, 18, 20, 21), and we introduce bulk masses $M_{e,i}$ and $M_{n,i}$. All the unwanted massless fields are made heavy by introducing appropriate brane fields and brane superpotentials, leaving only three sets of lepton-doublet chiral superfields, L_i, charged-lepton chiral superfields, E_i, and right-handed neutrino chiral superfields, N_i, below $1/R$. Introducing brane-localized Majorana mass terms for N_i's, we obtain the superpotential in Eq. (24), but now y_e, y_n and M_R are all 3×3 matrices. The Yukawa matrices, y_e and y_n, take similar forms to those of quarks, Eqs. (31, 32), while the right-handed neutrino Majorana mass matrix, M_R, has the most general structure.

3 Unified Theory: 5D $SU(6)$ Model

In this section we construct a unified version of the previous theory. The basic idea is the same. We consider a 5D supersymmetric gauge theory on S^1/Z_2, with non-trivial boundary conditions breaking the unified gauge symmetry. The low-energy theory below $1/R$ is a 4D $N = 1$ supersymmetric gauge theory with gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ (times an extra $U(1)$). The MSSM Higgs doublets arise from an extra dimensional component of the gauge field, *i.e.* the Σ field. Matter fields are introduced in the bulk, while the Yukawa couplings arise from the 5D gauge interaction. The various Yukawa couplings are controlled by bulk masses for the matter hypermultiplets, and the unwanted zero modes are all made heavy by introducing appropriate brane superfields and superpotentials. Unlike the previous $SU(3)_C \times SU(3)_W$ theory, however, this unified theory gives the correct normalization for hypercharges: the $SU(5)$ relation for the three MSSM gauge couplings. Therefore, assuming a large volume for the extra dimension, we recover the successful prediction of the MSSM for $\sin^2\theta_w$. The compactification scale is then given by the conventional unification scale, $1/R \sim 10^{16}$ GeV, at the leading order.

We first describe the gauge-Higgs sector of the theory. We consider a 5D supersymmetric $SU(6)$ gauge theory on S^1/Z_2. The boundary conditions for the 5D $SU(6)$ gauge multiplet is given as in Eq. (1) but with P and P' being 6×6 matrices. To break $SU(6)$ down to the standard model gauge group (with an extra $U(1)_X$ gauge group), we choose $P = \mathrm{diag}(1,1,1,1,1-1)$ and $P' = \mathrm{diag}(1,1,-1,-1,-1,-1)$. Specifically, the boundary conditions for the 5D gauge multiplet are written as

$$V: \left(\begin{array}{cc|ccc|c} (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) \\ (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) \\ \hline (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) \\ (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) \\ (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) \\ \hline (-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) \end{array}\right), \tag{36}$$

$$\Sigma: \left(\begin{array}{cc|ccc|c} (-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) \\ (-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) \\ \hline (-,+) & (-,+) & (-,-) & (-,-) & (-,-) & (+,-) \\ (-,+) & (-,+) & (-,-) & (-,-) & (-,-) & (+,-) \\ (-,+) & (-,+) & (-,-) & (-,-) & (-,-) & (+,-) \\ \hline (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) \end{array}\right), \tag{37}$$

where the first and second signs represent parities under the two reflections $Z: y \to -y$ and $Z': y' \to -y'$, respectively. Since only $(+,+)$ components have zero modes, we find from Eq. (36) that the 4D gauge symmetry below $1/R$ is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. Here we take $U(1)_Y$ as a $U(1)$ generator contained in the upper-left 5×5 block of the original 6×6 matrix. This implies that the upper-left 5×5 block is the conventional Georgi-Glashow $SU(5)$, and the standard model gauge group is embedded in it. Therefore, if the three MSSM gauge couplings arise entirely from the 5D bulk gauge coupling, we obtain the standard $SU(5)$ relation for them at the scale of $1/R$: $g_3 = g_2 = (5/3)^{1/2} g_Y$.

Now we consider gauge coupling unification in our theory in more detail. As we have discussed above, the 5D bulk gauge coupling, $\mathcal{L}_5 = (1/g^2) F_{MN}^2$, gives the $SU(5)$ relation for the MSSM gauge couplings. In general, however, the zero-mode gauge couplings also receive contributions from the brane-localized gauge kinetic operators, $\delta(y)\lambda_0 F_{\mu\nu}^2$ and $\delta(y-\pi R)\lambda_\pi F_{\mu\nu}^2$, which do not necessarily have to respect the $SU(5)$ relation (in our case, the operator at $y = \pi R$ does not respect it). Specifically, the zero-mode gauge couplings, g_0, are given by $1/g_0^2 = \pi R/g^2 + \lambda_0 + \lambda_\pi$, and thus are not exactly $SU(5)$ symmetric. Nevertheless, if the volume of the extra dimension is large compared with the cutoff scale of the theory, we expect that the zero-mode couplings are dominated by the bulk contribution, and the $SU(5)$ relation is recovered [8]. In particular, if we assume that the theory is strongly coupled at the cutoff scale, M_*, we can reliably estimate the size of various couplings using the naive dimensional analysis: $1/g^2 \simeq M_*/16\pi^3$ and $\lambda_0 \simeq \lambda_\pi \simeq 1/16\pi^2$, providing a reliable and predictive framework for gauge coupling unification in higher dimensions [14]. In our case, this leads to the standard $SU(5)$ relation for the gauge couplings, $g_3 = g_2 = (5/3)^{1/2} g_Y$, at the compactification scale, neglecting corrections from the logarithmic running of the gauge couplings between M_* and $1/R$. This determines the compactification scale to be around the conventional unification scale $1/R \sim 10^{16}$ GeV, at the leading order in the large logarithm $\ln(M_Z R)$.

How about the Higgs fields? From the boundary conditions for the Σ fields, Eq. (37), we find that the Σ field yields zero modes transforming as $(\mathbf{1},\mathbf{2})_{(1/2,6)} \otimes (\mathbf{1},\mathbf{2})_{(-1/2,-6)}$ under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ (here we have arbitrarily normalized the $U(1)_X$ charges). We identify these fields as the two Higgs doublets of the MSSM: $H_u \equiv \Sigma(\mathbf{1},\mathbf{2})_{(1/2,6)}$ and $H_d \equiv \Sigma(\mathbf{1},\mathbf{2})_{(-1/2,-6)}$. Since the theory is 4D $N=1$ supersymmetric below $1/R$, the Higgs quartic couplings arise from the D-term gauge potential.

Let us now discuss matter fields. The basic construction of the theory is quite similar to the previous $SU(3)_C \times SU(3)_W$ theory. For simplicity, here we discuss the structure of the $SU(6)$ theory for a single generation model, but the generalization to three generations is quite straightforward: we just have to introduce three copies of bulk and brane fields with general intergenerational mixings for the brane superpotentials.

We begin with the down-type quark. We introduce a hypermultiplet $\{\mathcal{D}, \mathcal{D}^c\}$ transforming as $\mathbf{15}$ of $SU(6)$. We choose the boundary conditions for this hypermultiplet as

$$\mathcal{D} = \mathcal{D}_Q^{(+,+)} \oplus \mathcal{D}_U^{(+,-)} \oplus \mathcal{D}_E^{(+,-)} \oplus \mathcal{D}_{\bar{D}}^{(-,-)} \oplus \mathcal{D}_{\bar{L}}^{(-,+)}, \tag{38}$$

$$\mathcal{D}^c = \mathcal{D}_{\bar{Q}}^{c\,(-,-)} \oplus \mathcal{D}_{\bar{U}}^{c\,(-,+)} \oplus \mathcal{D}_{\bar{E}}^{c\,(-,+)} \oplus \mathcal{D}_D^{c\,(+,+)} \oplus \mathcal{D}_L^{c\,(+,-)}, \tag{39}$$

where the superscripts denote transformation properties under (Z, Z'), and the subscripts represent the transformation properties of the component fields under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ as $Q : (\mathbf{3},\mathbf{2})_{(1/6,2)}$, $U : (\mathbf{3}^*,\mathbf{1})_{(-2/3,2)}$, $D : (\mathbf{3}^*,\mathbf{1})_{(1/3,4)}$, $L : (\mathbf{1},\mathbf{2})_{(-1/2,4)}$, $E : (\mathbf{1},\mathbf{1})_{(1,2)}$, $\bar{Q} : (\mathbf{3}^*,\mathbf{2})_{(-1/6,-2)}$, $\bar{U} : (\mathbf{3},\mathbf{1})_{(2/3,-2)}$, $\bar{D} : (\mathbf{3},\mathbf{1})_{(-1/3,-4)}$, $\bar{L} : (\mathbf{1},\mathbf{2})_{(1/2,-4)}$, and $\bar{E} : (\mathbf{1},\mathbf{1})_{(-1,-2)}$. We then find that zero modes arise only from $\mathcal{D}_Q$ and $\mathcal{D}_D^c$, which have the correct quantum numbers for the MSSM quark doublet, Q, and the down-type quark singlet, D, under $SU(3)_C \times SU(2)_L \times U(1)_Y$.

For the up-type quark, we introduce a hypermultiplet $\{\mathcal{U}, \mathcal{U}^c\}$ transforming as $\mathbf{20}$ of $SU(6)$. The boundary conditions are chosen as

$$\mathcal{U} = \mathcal{U}_{Q'}^{(+,+)} \oplus \mathcal{U}_{U'}^{(+,-)} \oplus \mathcal{U}_{E'}^{(+,-)} \oplus \mathcal{U}_{\bar{Q}'}^{(-,+)} \oplus \mathcal{U}_{\bar{U}'}^{(-,-)} \oplus \mathcal{U}_{\bar{E}'}^{(-,-)} \tag{40}$$

$$\mathcal{U}^c = \mathcal{U}_{\bar{Q}'}^{c\,(-,-)} \oplus \mathcal{U}_{\bar{U}'}^{c\,(-,+)} \oplus \mathcal{U}_{\bar{E}'}^{c\,(-,+)} \oplus \mathcal{U}_{Q'}^{c\,(+,-)} \oplus \mathcal{U}_{U'}^{c\,(+,+)} \oplus \mathcal{U}_{E'}^{c\,(+,+)}, \tag{41}$$

where the subscripts represent the transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ as $Q' : (\mathbf{3},\mathbf{2})_{(1/6,-3)}$, $U' : (\mathbf{3}^*,\mathbf{1})_{(-2/3,-3)}$, $E' : (\mathbf{1},\mathbf{1})_{(1,-3)}$, $\bar{Q}' : (\mathbf{3}^*,\mathbf{2})_{(-1/6,3)}$, $\bar{U}' : (\mathbf{3},\mathbf{1})_{(2/3,3)}$, and $\bar{E}' : (\mathbf{1},\mathbf{1})_{(-1,3)}$. We find that zero modes arise from $\mathcal{U}_{Q'}$, $\mathcal{U}_{U'}^c$ and $\mathcal{U}_{E'}^c$, of which the first two have the correct $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers for the MSSM quark doublet, Q, and the up-type quark singlet, U.

As in the model in the previous section, we introduce bulk masses for the hypermultiplets, M_u and M_d, which we here take $M_u < 0$ and $M_d < 0$. Then, we find that the zero modes for $\mathcal{U}_{Q'}$ and $\mathcal{D}_Q$ ($\mathcal{U}^c_{U'}$, $\mathcal{U}^c_{E'}$ and $\mathcal{D}^c_D$) are localized toward the $y=0$ ($y=\pi R$) brane. The 5D $SU(6)$ gauge interaction yields the superpotential Yukawa interaction

$$S = \int d^4x \int d^2\theta \; (y'_u \mathcal{U}_{Q'} \mathcal{U}^c_{U'} H_u + y'_d \mathcal{D}_Q \mathcal{D}^c_D H_d + \cdots) + \text{h.c.}, \tag{42}$$

where y'_u and y'_d are given by Eqs. (13, 14). These are still not the quark Yukawa couplings of the MSSM, since the "quark doublets", $\mathcal{U}_{Q'}$ and $\mathcal{D}_Q$, are different fields while they must be an identical field in the MSSM.

To reproduce the MSSM Yukawa couplings, we have to introduce brane-localized superfields and superpotentials at $y=0$. Since this brane possesses $SU(5) \times U(1)_X$, these fields and interactions must respect $SU(5) \times U(1)_X$. Defining $T_{\mathcal{D}} \equiv \mathcal{D}_Q \oplus \mathcal{D}_U \oplus \mathcal{D}_E$ and $T_{\mathcal{U}} \equiv \mathcal{U}_{Q'} \oplus \mathcal{U}_{U'} \oplus \mathcal{U}_{E'}$, which transform as $\mathbf{10}_2$ and $\mathbf{10}_{-3}$ under $SU(5) \times U(1)_X$ respectively, the required superpotential terms are written as

$$S = \int d^4x \; dy \; \delta(y) \left[\int d^2\theta \; \left\{ \bar{T}(\kappa_{T,1} T_{\mathcal{U}} + \kappa_{T,2} \bar{X} T_{\mathcal{D}}) + Y(X\bar{X} - \Lambda^2) \right\} + \text{h.c.} \right]. \tag{43}$$

Here, $\bar{T}$, X, $\bar{X}$, and Y are brane-localized chiral superfields transforming as $\mathbf{10}^*_3$, $\mathbf{1}_5$, $\mathbf{1}_{-5}$, and $\mathbf{1}_0$ under $SU(5) \times U(1)_X$, respectively; Λ is a mass scale which we assume to be $\sim 1/R$. This superpotential gives vacuum expectation values for the X and $\bar{X}$ fields, $\langle X \rangle = \langle \bar{X} \rangle = \Lambda$, breaking the $U(1)_X$ gauge symmetry. As a consequence, the mixing between $T_{\mathcal{U}}$ and $T_{\mathcal{D}}$ occurs. In particular, one linear combination of $\mathcal{U}_{Q'}$ and $\mathcal{D}_Q$, $Q_H \equiv \cos\phi_Q \, \mathcal{U}_{Q'} + \sin\phi_Q \, \mathcal{D}_Q$, becomes massive together with the $(\mathbf{3}^*, \mathbf{2})_{(-1/6,3)}$ component of $\bar{T}$; here $\tan\phi_Q = \kappa_{T,2}\langle \bar{X} \rangle / \kappa_{T,1}$. Therefore, at low energies, we have three quark chiral superfields, $Q \equiv -\sin\phi_Q \, \mathcal{U}_{Q'} + \cos\phi_Q \, \mathcal{D}_Q$, $U \equiv \mathcal{U}^c_{U'}$ and $D \equiv \mathcal{D}^c_D$, which we identify as the MSSM quark superfields. The low-energy Yukawa couplings are given by

$$S = \int d^4x \int d^2\theta \; (y_u Q U H_u + y_d Q D H_d) + \text{h.c.}, \tag{44}$$

where $y_u = -y'_u \sin\phi_Q$ and $y_d = y'_d \cos\phi_Q$. The unwanted zero mode from $\mathcal{U}^c_{E'}$ is made heavy by introducing a brane-localized field $\bar{C}$ at $y = \pi R$, transforming as $(\mathbf{4}^*, \mathbf{1})_{-3}$ under the brane gauge group $SU(4)_C \times SU(2)_L \times U(1)$, and the superpotential $\delta(y - \pi R)[\bar{C} C_{\mathcal{U}}]_{\theta^2}$, where $C_{\mathcal{U}} \equiv \mathcal{U}^c_{\bar{U}'} \oplus \mathcal{U}^c_{E'}$. We thus recover the quark sector of the MSSM below the scale of $1/R \sim \Lambda$.

The lepton sector can be worked out similarly. For the charged lepton, we introduce a hypermultiplet $\{\mathcal{E}, \mathcal{E}^c\}$ transforming as $\mathbf{15}$ of $SU(6)$. The boundary conditions are chosen as

$$\mathcal{E} = \mathcal{E}_Q^{(+,-)} \oplus \mathcal{E}_U^{(+,+)} \oplus \mathcal{E}_E^{(+,+)} \oplus \mathcal{E}_{\bar{D}}^{(-,+)} \oplus \mathcal{E}_{\bar{L}}^{(-,-)}, \tag{45}$$

$$\mathcal{E}^c = \mathcal{E}_{\bar{Q}}^{c\,(-,+)} \oplus \mathcal{E}_{\bar{U}}^{c\,(-,-)} \oplus \mathcal{E}_{\bar{E}}^{c\,(-,-)} \oplus \mathcal{E}_D^{c\,(+,-)} \oplus \mathcal{E}_L^{c\,(+,+)}. \tag{46}$$

The zero modes arise from $\mathcal{E}_U$, $\mathcal{E}_E$ and $\mathcal{E}^c_L$, of which the last two have the correct quantum numbers for the MSSM charged lepton, E, and the lepton doublet, L, under $SU(3)_C \times SU(2)_L \times U(1)_Y$. We also introduce a hypermultiplet $\{\mathcal{N}, \mathcal{N}^c\}$ transforming as $\mathbf{6}$ of $SU(6)$, with the following boundary conditions:

$$\mathcal{N} = \mathcal{N}^{(-,+)}_{\bar{D}'} \oplus \mathcal{N}^{(-,-)}_{\bar{L}'} \oplus \mathcal{N}^{(+,+)}_{N}, \tag{47}$$

$$\mathcal{N}^c = \mathcal{N}^{c\,(+,-)}_{D'} \oplus \mathcal{N}^{c\,(+,+)}_{L'} \oplus \mathcal{N}^{c\,(-,-)}_{\bar{N}}, \tag{48}$$

where the subscripts represent the transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ as $D' : (\mathbf{3}^*, \mathbf{1})_{(1/3,-1)}$, $L' : (\mathbf{1}, \mathbf{2})_{(-1/2,-1)}$, $N : (\mathbf{1}, \mathbf{1})_{(0,-5)}$, $\bar{D}' : (\mathbf{3}, \mathbf{1})_{(-1/3,1)}$, $\bar{L}' : (\mathbf{1}, \mathbf{2})_{(1/2,1)}$, and $\bar{N} : (\mathbf{1}, \mathbf{1})_{(0,5)}$. This gives the zero-mode fields from $\mathcal{N}_N$ and $\mathcal{N}^c_{L'}$. Introducing bulk hypermultiplet masses with $M_e, M_n > 0$, the zero-mode fields $\mathcal{E}_U$, $\mathcal{E}_E$ and $\mathcal{N}_N$ ($\mathcal{E}^c_L$ and $\mathcal{N}^c_{L'}$) are localized toward the $y = \pi R$ ($y = 0$) brane. The 5D $SU(6)$ gauge interaction then leads to the superpotential couplings

$$S = \int d^4x \int d^2\theta \; (y'_e \mathcal{E}^c_L \mathcal{E}_E H_d + y'_n \mathcal{N}^c_{L'} \mathcal{N}_N H_u) + \text{h.c.}, \tag{49}$$

where y'_e (y'_n) is given by Eq. (13) with $y'_u \to y'_e$ and $M_u \to M_e$ ($y'_u \to y'_n$ and $M_u \to M_n$).

The brane interactions are given as follows. We first introduce a brane field $\bar{A}$ at $y = \pi R$, transforming as $(\mathbf{6}, \mathbf{1})_2$ under the brane gauge group $SU(4)_C \times SU(2)_L \times U(1)$, and the superpotential $\delta(y - \pi R)[\bar{A} A_{\mathcal{E}}]_{\theta^2}$, where $A_{\mathcal{E}} \equiv \mathcal{E}_U \oplus \mathcal{E}_{\bar{D}}$. This makes an unwanted zero-mode field from $\mathcal{E}_U$ heavy together with the $(\mathbf{3}, \mathbf{1})_{(2/3,-2)}$ component of $\bar{A}$. We next define $F_{\mathcal{E}^c} \equiv \mathcal{E}^c_D \oplus \mathcal{E}^c_L$ and $F_{\mathcal{N}^c} \equiv \mathcal{N}^c_{D'} \oplus \mathcal{N}^c_{L'}$, which transform as $\mathbf{5}^*_4$ and $\mathbf{5}^*_{-1}$ under $SU(5) \times U(1)_X$, respectively. Introducing a brane-localized field $\bar{F}$ at $y = 0$, which transforms as $\mathbf{5}_{-4}$, we write the brane superpotential terms

$$S = \int d^4x \; dy \; \delta(y) \left[\int d^2\theta \; \bar{F} (\kappa_{F,1} F_{\mathcal{E}^c} + \kappa_{F,2} X F_{\mathcal{N}^c}) + \text{h.c.} \right]. \tag{50}$$

Then a linear combination of $\mathcal{E}^c_L$ and $\mathcal{N}^c_{L'}$, $L_H \equiv \cos\phi_L \, \mathcal{E}^c_L + \sin\phi_L \, \mathcal{N}^c_{L'}$, becomes massive together with the $(\mathbf{1}, \mathbf{2})_{(1/2,-4)}$ component of $\bar{F}$; here $\tan\phi_L = \kappa_{F,2} \langle X \rangle / \kappa_{F,1}$. Thus, at low energies, we have three lepton chiral superfields, $L \equiv -\sin\phi_L \, \mathcal{E}^c_L + \cos\phi_L \, \mathcal{N}^c_{L'}$, $E \equiv \mathcal{E}_E$ and $N \equiv \mathcal{N}_N$, which we identify as the MSSM lepton superfields and the right-handed neutrino superfield. The Yukawa couplings for them are given by

$$S = \int d^4x \int d^2\theta \; (y_e L E H_d + y_n L N H_u) + \text{h.c.}, \tag{51}$$

where $y_e = -y'_e \sin\phi_L$ and $y_n = y'_n \cos\phi_L$.

In the present scenario, there are two possibilities for obtaining small neutrino masses. In the case where M_n is sizable, it is difficult to implement the conventional high-scale seesaw

	V	$\Sigma \subset H, \bar{H}$	M	M^c	B	X	$\bar{X}$	Y
$U(1)_R$	0	0	1	1	1	0	0	2

Table 1: $U(1)_R$ charges for 4D vector and chiral superfields; $\{M, M^c\}$ represent bulk matter hypermultiplets, while B represents brane-localized matter fields that mix with the bulk matter.

mechanism because the wave-function for the right-handed neutrino is exponentially suppressed at the $y = 0$ brane where $U(1)_X$ is broken. In this case, however, we can consider an alternative way of obtaining small neutrino masses. We simply assume that the bulk mass M_n is somewhat larger compared with the other bulk masses ($M_n \gg 1/R$). The neutrino Yukawa coupling, y_n, is then exponentially suppressed, $y_n \sim \exp(-\pi R|M_n|)$, and we naturally obtain a small Dirac neutrino mass. Together with the quark sector, we find that the theory below $1/R$ reduces to the MSSM with the right-handed neutrino, where small Dirac neutrino masses are obtained by tiny neutrino Yukawa couplings. The second possibility is that M_n is small compared with the compactification scale ($M_n \lesssim 1/R$). This is possible because there is no direct experimental constraint for the neutrino Yukawa couplings so that their size can be similar to that of the gauge couplings. Since the wave-function for the right-handed neutrino is nearly flat in this case, we can give a large Majorana mass for the right-handed neutrino by introducing the superpotential term $\delta(y)[X^2 \mathcal{N}_N^2]_{\theta^2}$, leading to a small Majorana neutrino mass for the observed left-handed neutrino through the seesaw mechanism. Therefore, the theory below $1/R$ is the MSSM with small neutrino masses arising from the conventional seesaw mechanism. It is also interesting to point out that the resulting light neutrino masses are not expected to exhibit a strong hierarchy due to the irrelevance of the bulk mass parameters that could potentially generate it.

Finally, we comment on the R symmetry structure of the theory. As stressed in Refs. [8, 14], higher dimensional theories naturally possess a special $U(1)_R$ symmetry which forbids dangerous dimension four and five proton decay operators. The $U(1)_R$ charges are given such that the gauge and Higgs fields, V and H, have vanishing charges while the matter fields, M, have charges of $+1$. In our theory, this $U(1)_R$ symmetry arises simply as a $U(1)$ subgroup of the $SU(2)_R$ automorphism group of the 5D supersymmetry algebra. The explicit $U(1)_R$ charge assignment is given in Table 1. Requiring the $U(1)_R$ symmetry for the entire theory, dangerous dimension four and five proton decay operators are completely forbidden. After 4D $N = 1$ supersymmetry is broken, this $U(1)_R$ symmetry will be broken, presumably to the Z_2 R-parity subgroup. Since the breaking scale is small, however, it does not reintroduce the problem of proton decay.

4 Conclusions and Discussion

We have considered the unification of the Higgs and gauge fields in higher dimensions: the Higgs fields arise from extra dimensional components of higher dimensional gauge fields. To incorporate the Higgs doublets in an adjoint representation, the original higher dimensional gauge group must be larger than the standard model gauge group. This larger gauge symmetry is then broken to the standard model one at low energies by orbifold compactifications. Previous work along this direction had encountered several difficulties. In particular, it is generically difficult to obtain both a sufficiently large quartic coupling for the Higgs fields and a realistic structure for the Yukawa couplings, due to higher dimensional gauge invariance which acts non-linearly on the Higgs fields.

In this paper we have constructed realistic theories in which (a part of) the Higgs fields are identified with extra dimensional components of the gauge field. There are two ways to obtain the quartic coupling in the low-energy Higgs potential: from 6D gauge kinetic energies and from supersymmetric D-term potential. In this paper we have adopted the latter, which allows us to consider 5D theories. We have constructed both a minimal version (5D $SU(3)_C \times SU(3)_W$ model) and a unified version (5D $SU(6)$ model) of the theory. While there is no prediction for the observed gauge couplings and the value of $1/R$ in the minimal theory, the unified theory gives the successful MSSM prediction for $\sin^2\theta_w$ and $1/R \sim 10^{16}$ GeV, if the volume of the extra dimension is taken to be large.

The Yukawa couplings are generated from the higher dimensional gauge interaction by introducing matter fields in the bulk. Working out the boundary conditions carefully, we obtain all the MSSM Yukawa couplings at low energies. Although they arise from higher dimensional gauge interactions, the sizes of these Yukawa couplings can be different from the 4D gauge couplings due to the suppression factors coming from wave-function profiles of the matter zero modes determined by bulk mass parameters. Unwanted massless fields are all made heavy by introducing appropriate matter and superpotentials on branes. This bulk-brane mixing is also the source of intergenerational mixings in the low-energy Yukawa matrices. Small neutrino masses are accommodated in the theory either through the conventional seesaw mechanism or through small Dirac neutrino Yukawa couplings arising from exponential wave-function suppressions. It is remarkable that we can obtain a completely realistic Yukawa structure with bulk matter fields without conflicting with higher dimensional gauge invariance; if we put matter on a brane, as is often considered in literature, higher dimensional gauge invariance forbids local Yukawa couplings on a brane.

Below the compactification scale, our theory is reduced almost to the MSSM. We have, however, not specified how we obtain the supersymmetric mass term (μ term) of the Higgs doublets.

There are a number of ways to generate the μ term supersymmetrically; for example, we can introduce additional bulk fields Φ coupled to the Higgs fields and generate a term connecting H_u and H_d by integrating out some of the Φ fields, inducing the μ term by giving the remaining Φ fields vacuum expectation values. However, these are generically quite complicated and not satisfactory. A more interesting possibility is that the μ term arises through supersymmetry breaking. Here we present two simple cases where the μ term is generated naturally. First, we observe that the 4D Kähler potential of the theory contains the term $[\Sigma^2]_{\theta^2\bar{\theta}^2} \supset [H_u H_d]_{\theta^2\bar{\theta}^2}$, which consists of the 5D gauge kinetic term. After supersymmetry is broken, this term leads to a μ term of the order of the gravitino mass $m_{3/2}$, through supergravity effects [15]. [This is due to the fact that the supergravity theory has the symmetry $(K, W) \rightarrow (K - f, W e^{f/M_{\rm Pl}^2})$, where K and W are the Kähler and superpotentials. Thus, the above term can be transferred to the superpotential as $W e^{H_u H_d/M_{\rm Pl}^2}$. Considering $\langle W \rangle \simeq m_{3/2} M_{\rm Pl}^2$ to cancel the cosmological constant, we find that the μ term, $W \simeq m_{3/2} H_u H_d$, is generated.] Therefore, if the gravitino mass is of order the weak scale, as in the supergravity mediation scenario, we naturally obtain the correct size of the μ term. Another scenario where the μ term arises naturally is one in which supersymmetry is broken by boundary conditions imposed on the extra dimension [16] (or by the F-term expectation value for the radion superfield [17, 18]). In this case the gaugino masses arise from the twisting of boundary conditions by $U(1) \subset SU(2)_R$ under the orbifold translation $y \rightarrow y + 2\pi R$. In our theories, however, the Higgsinos are "gauginos" in higher dimensions, so that the Higgsino mass (*i.e.* μ term) is also generated by this twisting. We also find that the resulting squark and slepton masses are universal, regardless of the presence of the bulk hypermultiplet masses and brane-bulk mixings. Therefore, this scenario can lead to a realistic phenomenology at low energies. In particular, in the case where contributions from brane-localized kinetic operators are negligible, we obtain the prediction $m_{1/2} \equiv \tilde{m}$, $m_{\tilde{q},\tilde{l}}^2 = \tilde{m}^2$, $m_{h_u,h_d}^2 = -\tilde{m}^2$, $\mu = -\tilde{m}$, $B = 0$ and $A = -\tilde{m}$ at the compactification scale $1/R$, where $m_{1/2}$, $m_{\tilde{q},\tilde{l}}$, m_{h_u,h_d}, μB, and A are the universal gaugino mass, the universal squark and slepton mass, the Higgs soft mass, the holomorphic supersymmetry-breaking mass for the Higgs doublets, and the trilinear scalar coupling, respectively. Incidentally, the supersymmetric CP problem is absent in this scenario, since all supersymmetry breaking parameters can be made real at the compactification scale.

While we have not used the higher dimensional origin of the Higgs to regulate the quadratic divergence of the Higgs boson (it is still regulated by supersymmetry), we think that understanding the origin of the Higgs fields constitutes a significant advance in terms of the unification program. In particular, in our $SU(6)$ unified model, all the MSSM gauge fields *and* the Higgs doublets are unified into a single 5D gauge multiplet; both gauge symmetry breaking and extraction of the Higgs fields are attained by boundary conditions imposed on a single extra dimension.

In our framework, the original higher dimensional theory contains only the gauge multiplet and chiral matter fields, and the Higgs fields arise from the gauge multiplet. This sheds light on the famous question: why we have light vector-like Higgs doublets in the MSSM despite the absence of a symmetry protecting their mass? Our answer is: because they are gauge fields. Higher dimensional gauge invariance forbids a mass term for the Higgs fields, and once forbidden it is not generated due to the supersymmetric non-renormalization theorem. The required μ term of the order of the weak scale will be generated through compactification and, possibly, supersymmetry breaking effects. It will be interesting to further explore phenomenological consequences of the models, especially related to the supersymmetry breaking mechanisms discussed in the previous paragraph.

Note added:

While preparing this manuscript, we received Ref. [19] which considers the Higgs field arising from extra dimensional components of gauge fields in a different context.

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